

Adaptive Neural Network Control of a Self-balancing Two-wheeled Scooter

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Abstract—This paper presents an adaptive neural network control for a two-wheeled self-balancing scooter for pedagogical purposes. A mechatronic system structure driven by two DC motors is described, and its mathematical modeling incorporating the friction between the wheels and motion surface is derived. By decomposing the overall system into two subsystems: rotation and inverted pendulum, we design two adaptive radial-basis-function (RBF) neural network (DOF) controllers to achieve self-balancing and rotation control. Experimental results indicate that the proposed controllers are capable of providing appropriate control actions to steer the vehicle in desired manners.

Keywords: adaptive neural network control, digital signal processing, gyroscope, inverted pendulum, robotics transporter.

I. INTRODUCTION

Recently, self-balancing two-wheeled transporters, like the SegwayTM [1] and PMP [2], have been well recognized as powerful personal transportation vehicles. The kind of transporter can be usually constructed by a synthesis of mechatronics, control techniques and software. For example, the SegwayTM is made by quite high-tech and high-quality dedicated components, such brushless servomotor with neodymium magnets, precision gearbox, NiMH batteries, silica-based wheels, a digital signal processor as a main controller, motor drivers, six gyroscopes, and several safety accessories. The author in [3] compared several kinds of vehicles and proposed that the SegwayTM is a good two-wheeled vehicle for community patrol. In contrast to the SegwayTM, many researchers [5]-[10] presented low-tech self-balancing transporters and claimed that the vehicle can be built using the off-the-shelf inexpensive components. With the advent of modern technology, such transporters with sophisticated safety features can be cost down so that they, like traditional bicycles, have highly potential to become prevalent two-wheeled scooters, satisfying human transportation requirements.

Design and implementation of a safe and practical self-balancing scooter is a very interesting problem. This problem has attracted much attention in recent years. Sasaki [2] constructed a lightweight self-balancing personal riding-type wheeled mobile platform (PMP); the PMP steering control was achieved by changing the position of the rider's center of gravity. Grasser *et al.* [4] presented an unmanned mobile inverted pendulum, and Kaustubh *et al.* [5], studied the dynamic equations of the wheeled inverted pendulum by partial feedback linearization. However, they are test prototypes, aiming at providing several theoretical design and

analytical approaches. The above-mentioned survey reveals that little attention has been paid to design a pragmatic and safe self-balancing scooter. The goal of this paper is to construct an experimental two-wheeled self-balancing scooter with low-cost and low-tech components and propose two adaptive neural network controllers for achieving self-balancing and rotation.

Comparing with the design presented by Grasser *et al.* [4], one finds that the key features of the proposed system design hinge on the theoretical development of its mathematical modeling with frictions, and adaptive control of the scooter. Like the JOE in [4], the vehicle system can be decoupled into two subsystems: rotation and inverted pendulum. Based on the idea, we design two adaptive RBF NN controllers for achieving rotation control and self-balancing. For details of RBF NN, the reader is referred to [9] and therein. The proposed adaptive RBF NN control methods are useful and powerful in keeping the almost same driving performance for different riders.

The rest of the paper is outlined as follows. Section II briefly describes the mechatronic design, control architecture, sensing, signal conditioning and mathematical modeling. The mathematical modeling of the scooter is derived in Section III. Section IV is devoted to developing the adaptive two-DOF controller for the decoupled self-balancing and the adaptive PD controller with a prefilter for rotation control. In Section V several experiments are conducted to show the feasibility and effectiveness of the proposed control methods. Section VI concludes the paper.

II. BRIEF SYSTEM DESIGN

2.1 Mechatronic Design

Fig.1 displays the photograph of the laboratory-based personal two-wheeled scooter with differential driving. This vehicle is composed of one foot plate, two 24V DC motors with gearbox and two stamped steel wheels with 14" tires, two 12-volt sealed rechargeable lead-acid batteries in series, two motor drivers, one digital signal processor (DSP) TI 320F240 from Texas Instrument as a main controller, one handle-bar with a potentiometer as a position sensor, one gyroscope and one tilt sensor. The two motor drivers use dual H-bridge circuitry to deliver PWM power to drive the two DC servomotors. Sending PWM signals to the H-bridge circuit, the DSP controls linear speed and rotation of the scooter as well as maintains the balance of the scooter. The gyroscope

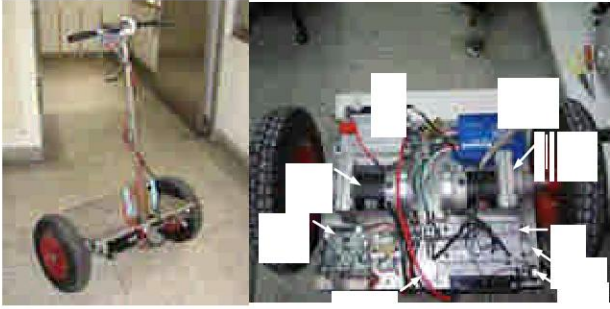


Fig.1. Laboratory-built personal two-wheeled scooter, (a) front view (b)Bottom view

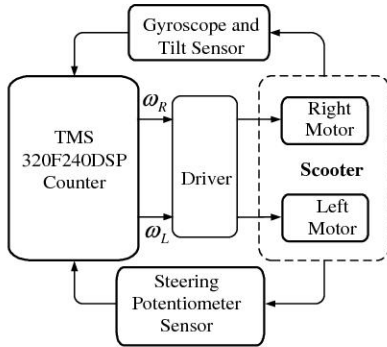


Fig. 2. Control Architecture.

and the tilt sensor are employed for measuring the rate and the angle of the inclination of the footplate caused by the rider. The two rechargeable lead-acid batteries directly provide power for the two DC servomotors and drivers, and the controller and all the sensors via DC-DC buck conversion.

2.2 Control Architecture

Fig.2 illustrates the block diagram of the entire scooter control system. The DSP controller with built-in A/D converter is responsible for executing the control algorithms including rotation control and self-balancing control. The feedback signals from the gyroscope and the tilt sensor are utilized via the controller to maintain the human body on the footplate without falling. The working principle of the self-balancing control is simply interpreted as below. If the user leans forward, the vehicle will move forward in order to maintain the human body without falling. The signal taken from the potentiometer is used in the controller to rotate the scooter to the desired angle.

2.3 Sensing and Signal Conditioning

The subsection describes all the sensors and their signal conditioning in Fig. 2. The pitch angle rate reading ω_p from the gyroscope and the pitch angle reading θ_p from one tilt sensor are processed by two first-order filters filtering, thus reducing the noise effects in the best minimum mean-squared error sense. The potentiometer is adopted to measure the shaft

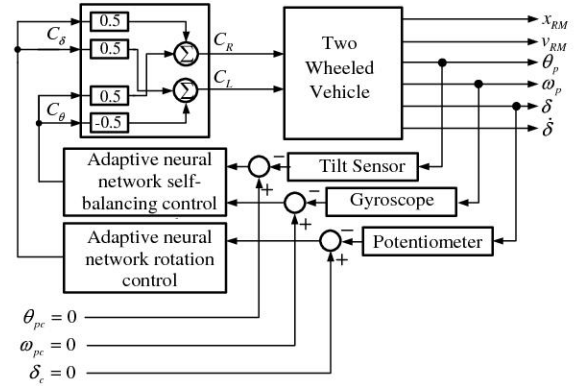


Fig.3. Block diagram of the scooter controller.

angle of the handlebar and the position signal is directly read by the DSP controller with necessary signal processing.

2.4 MATHEMATICAL MODELING

In designing a controller to steer the vehicle, one needs to develop its mathematical model such that the controller can then be designed to achieve the desired control objective. The nonlinear mathematical model of the scooter with the frictional force is well described in [11] and its linearized system is given in the following state-space form

$$\begin{bmatrix} \dot{x}_{RM} \\ \dot{v}_{RM} \\ \dot{\theta}_p \\ \dot{\omega}_p \\ \dot{\delta} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -A_{22} & A_{23} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & A_{43} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \begin{bmatrix} x_{RM} \\ v_{RM} \\ \theta_p \\ \omega_p \\ \delta \\ \delta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_2 & -B_2 \\ 0 & 0 \\ B_4 & B_4 \\ 0 & 0 \\ B_6 & -B_6 \end{bmatrix} \begin{bmatrix} C_L \\ C_R \end{bmatrix} - \begin{bmatrix} \bar{f}_1 \\ \bar{f}_2 \\ \bar{f}_3 \\ \bar{f}_4 \\ \bar{f}_5 \\ \bar{f}_6 \end{bmatrix} \quad (1)$$

where x_{RM} [m] and v_{RM} [m/s] respectively denote the position and velocity of the scooter; θ_p [rad] and ω_p [rad/s] respectively represent the pitch angle and pitch angle rate of the scooter; δ [rad] and $\dot{\delta}$ [rad/s] respectively stand for the yaw angle, yaw angle rate of the scooter. C_L [Nm] and C_R [Nm] respectively denote the applied torques on the left and right wheels.

Since the system model (1) is similar to that of the special vehicle, called JOE, developed by Grasser *et al.*, a well-known decoupling control approach shown in [4] can be applied to achieve the goal, thereby simplifying the controller design. Hence, the following decoupling transformation from C_θ and C_δ into the wheel torques C_R and C_L is used to decouple the system model such that the rotation control and inverted pendulum control are independently designed.

$$\begin{pmatrix} C_L \\ C_R \end{pmatrix} = \begin{pmatrix} 0.5 & 0.5 \\ 0.5 & -0.5 \end{pmatrix} \begin{pmatrix} C_\theta \\ C_\delta \end{pmatrix} \quad (2)$$

This transformation (2) converts the system model (1) into two subsystems; one is the inverted pendulum subsystem described by

$$\begin{pmatrix} \dot{\theta}_p \\ \dot{\omega}_p \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ A_{43} & 0 \end{pmatrix} \begin{pmatrix} \theta_p \\ \omega_p \end{pmatrix} + \begin{pmatrix} 0 \\ B_4 \end{pmatrix} C_\theta - \begin{pmatrix} 0 \\ f_4 \end{pmatrix} \quad (3)$$

and the other is the rotation subsystem described by

$$\begin{pmatrix} \dot{\delta} \\ \dot{\ddot{\delta}} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & A_{66} \end{pmatrix} \begin{pmatrix} \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ B_6 \end{pmatrix} C_\delta - \begin{pmatrix} 0 \\ \bar{f}_6 \end{pmatrix} \quad (4)$$

III. Adaptive Backstepping control Using RBF

3.1 Backstepping Self-Balancing Control Using RBF NN

Consider that the inverted pendulum subsystem (4) which is equivalent to the following equation

$$\begin{pmatrix} \dot{\theta}_p \\ \dot{\omega}_p \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ A_{43} & 0 \end{pmatrix} \begin{pmatrix} \theta_p \\ \omega_p \end{pmatrix} + \begin{pmatrix} 0 \\ B_4 \end{pmatrix} (C_\theta - \bar{f}^*) \quad (5)$$

where $\bar{f}^* = \bar{f}_4 / B_4$. Furthermore, we rewrite (5) as

$$\begin{pmatrix} \dot{\theta}_p \\ \dot{\omega}_p \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ A_{43} & 0 \end{pmatrix} \begin{pmatrix} \theta_p \\ \omega_p \end{pmatrix} + \begin{pmatrix} 0 \\ B_4 \end{pmatrix} \bar{f} \quad \text{where } \bar{f} = C_\theta - \bar{f}^* \quad (6)$$

where A_{43} and B_4 are two unknown parameters. The friction \bar{f}^* , including the Coulomb friction and the static friction, can be approximated using the RBF NN, that is, there exists the following function approximation:

$$|\bar{f}^*| \leq \bar{f}_{\max}^* = W^T S + \varepsilon = [w_1 \cdots w_n] [s_1 \cdots s_n]^T + \varepsilon \quad (7)$$

with W being the optimal weight vector. An inequality is imposed on the approximation error ε in order to give it an upper bound

$$|\varepsilon| \leq \varepsilon_d$$

Moreover, the variable $s_i, i = 1, 2, 3, \dots, n$, used for the RBF NN is the Gaussian functions defined by

$$s_i = \exp\left[-\frac{(x - u_{1i})^2}{\sigma_i^2} - \frac{(\dot{x} - u_{2i})^2}{\sigma_i^2}\right] \quad (8)$$

where u_{1i} and u_{2i} are the center of the receptive field and σ_i is the width of the Gaussian function. To achieve control of system (6), we define the first state variable θ_1 as the angle position θ_p and the second state variable θ_2 as the angle rate ω_p . The control objective is to control the angle position θ_1 to reach to the command position θ_{com} without error. Due to the nonlinearity of system (4), the well-known backstepping approach is employed to accomplish the control goal. The design procedure is stated in the following. To maintain the angle position θ_1 at $\theta_{com} = 0$, one defines the tracking error

$$\tilde{\theta}_1 = \theta_1 - \theta_{com} \quad (9)$$

Taking the time derivative of $\tilde{\theta}_1$ yields

$$\dot{\tilde{\theta}}_1 = \dot{\theta}_1 - \dot{\theta}_{com} = \dot{\theta}_1 = \theta_2 \quad (10)$$

The dynamics of the tracking error $\tilde{\theta}_1$ can be stabilized by choosing

$$\theta_2 = -K_1 \tilde{\theta}_1 \quad (11)$$

such that

$$\dot{\tilde{\theta}}_1 = -K_1 \tilde{\theta}_1, \quad K_1 > 0 \quad (12)$$

which indicates that $\tilde{\theta}_1$ will tend to zero as t approaches infinity. This can be easily proven by choosing the following Lyapunov function

$$V_1 = \tilde{\theta}_1^2 / 2 \quad (13)$$

which leads to

$$\dot{V}_1 = \tilde{\theta}_1 \dot{\tilde{\theta}}_1 = -K_1 \tilde{\theta}_1^2 < 0 \quad (14)$$

Next, we define a backstepping error ξ as follows

$$\xi = \theta_2 - (-K_1 \tilde{\theta}_1) = \theta_2 + K_1 \tilde{\theta}_1 \quad (15)$$

The time derivative of ξ can be written as

$$\dot{\xi} = \dot{\theta}_2 + K_1 \dot{\tilde{\theta}}_1 = A_{43}(\tilde{\theta}_1 + \theta_{com}) + B_4(C_\theta - \bar{f}^*) + K_1 \theta_2 \quad (16)$$

where \bar{f}^* is bounded.

To stabilize the system (16), we propose the following torque control:

$$C_\theta = (W^T S + K^*) \text{sgn}(\xi) - [K_p \tilde{\theta}_1 + A_{43} \theta_{com} + K_1 \theta_2] / B_4 \quad (17)$$

where $K_p > A_{43}$

Substituting the torque control (17) into (16), the time derivative of backstepping error is as follows

$$\dot{\xi} = (A_{43} - K_p) \tilde{\theta}_1 + B_4 (W^T S + K^*) \text{sgn}(\xi) - B_4 \bar{f}^* \quad (18)$$

To prove the asymptotic stability of the closed-loop system (18), we choose the following Lyapunov function candidate

$$V_2 = (K_p - A_{43}) \tilde{\theta}_1^2 / 2 + \xi^2 / 2 \quad (19)$$

The time derivative of V_2 along its motion trajectory of the system (18) is given by

$$\begin{aligned} \dot{V}_2 &= (K_p - A_{43}) \tilde{\theta}_1 \dot{\tilde{\theta}}_1 - K_1 (K_p - A_{43}) \tilde{\theta}_1^2 + \xi (A_{43} - K_p) \tilde{\theta}_1 + B_4 (W^T S + K^*) |\xi| \\ &\quad - B_4 \xi \bar{f}^* \leq -K_1 (K_p - A_{43}) \tilde{\theta}_1^2 + B_4 (W^T S + K^*) |\xi| - B_4 |\xi| \bar{f}_{\max}^* \\ &= -K_1 (K_p - A_{43}) \tilde{\theta}_1^2 + B_4 |\xi| (K^* - \varepsilon) \leq 0 \end{aligned} \quad (20)$$

provided that $K_p > A_{43}, \varepsilon > K^*, B_4 > 0$. Since \dot{V}_2 is negative semidefinite, it implies that $\xi \rightarrow 0$ and $\tilde{\theta}_1 \rightarrow 0$ as $t \rightarrow \infty$. The following theorem summarizes the result.

Theorem 1 Consider the inverted pendulum system (6) with the proposed backstepping control (17). Then the origin is globally asymptotically stable, i.e., $\xi \rightarrow 0$ and $\tilde{\theta}_1 \rightarrow 0$ as $t \rightarrow \infty$, i.e., $\theta_p \rightarrow \theta_{com}$, and $\dot{\theta}_p \rightarrow \dot{\theta}_{com}$ as $t \rightarrow \infty$.

3.2 Adaptive Backstepping Self-Balancing Control Using RBF NN

The main objective of the adaptive control law for the scooter is to determine a stable adaptive law with the parameter adjustment rules. The adaptive control law is proposed as follows:

$$C_\theta = (\hat{W}^T S + \hat{\varepsilon}) \text{sgn}(\xi) - [K_p \tilde{\theta}_1 + \hat{A}_{43} \theta_{com} + K_1 \theta_2] / \hat{B}_4 \quad (21)$$

where K_p and K_1 are two fixed constants, and the parameter adaptation laws for \tilde{A}_{43} , $\hat{\varepsilon}$, \tilde{B}_4 and \tilde{W} are given by

$$\dot{\tilde{B}}_4 = (r_b \xi / \hat{B}_4) (-K_p \tilde{\theta}_1 - K_1 \theta_2 - \hat{A}_{43} \theta_{com}) \quad (22)$$

$$\dot{\tilde{W}} = r_w S |\xi| \quad (23)$$

$$\dot{\hat{\varepsilon}} = 2r_\varepsilon |\xi| \quad (24)$$

$$\dot{\tilde{A}}_{43} = r_a \xi \theta_{com} \quad (25)$$

In what follows shows the way to derive the parameter adaptation laws (22)–(25). Define the backstepping error ξ by

$$\xi = \theta_2 - (-K_1 \tilde{\theta}_1) = \theta_2 + K_1 \tilde{\theta}_1 \quad (26)$$

The substitution of (21) into the system (16) yields

$$\begin{aligned} \dot{\xi} = & B_4 [\tilde{W}^T S \text{sgn}(\xi) + \hat{\varepsilon} \text{sgn}(\xi) - \bar{f}^*] + [1 - (B_4 / \hat{B}_4)] K_1 \theta_2 \\ & + [A_{43} - (B_4 / \hat{B}_4) K_p] \tilde{\theta}_1 + [A_{43} - (B_4 / \hat{B}_4) \hat{A}_{43}] \theta_{com} \end{aligned} \quad (27)$$

The errors of the adaptive parameters \tilde{A}_{43} , \tilde{B}_4 and \tilde{W} are expressed as follows

$$\tilde{A}_{43} = A_{43} - \hat{A}_{43} \Rightarrow \dot{\tilde{A}}_{43} = -\dot{\hat{A}}_{43} \quad (28)$$

$$\tilde{B}_4 = B_4 - \hat{B}_4 \Rightarrow \dot{\tilde{B}}_4 = -\dot{\hat{B}}_4 \quad (29)$$

$$\tilde{W} = W - \hat{W} \Rightarrow \dot{\tilde{W}} = -\dot{\hat{W}} \quad (30)$$

To show the asymptotical stability of the system, a Lyapunov function candidate is chosen as

$$\begin{aligned} V_3 = & (K_p - A_{43}) \tilde{\theta}_1^2 / 2 + \xi^2 / 2 + \tilde{A}_{43}^2 / 2r_a + \tilde{B}_4^2 / 2r_b \\ & + B_4 \tilde{W}^T \tilde{W} / 2r_w + B_4 (\varepsilon_d - \hat{\varepsilon})^2 / 2r_\varepsilon \end{aligned} \quad (31)$$

Taking derivative of the Lyapunov candidate yields

$$\begin{aligned} \dot{V}_3 \leq & -(K_p - A_{43}) K_1 \tilde{\theta}_1^2 + B_4 \tilde{W}^T (S |\xi| - \dot{\hat{W}} / r_w) - B_4 |\xi| \tilde{\varepsilon} \\ & + \tilde{B}_4 (-K_p \tilde{\theta}_1 \xi / \hat{B}_4 - \dot{\hat{B}}_4 / r_b - K_1 \xi^2 / \hat{B}_4 - K_1^2 \tilde{\theta}_1 \xi / \hat{B}_4) \\ & - \tilde{B}_4 (\hat{A}_{43} \xi \theta_{com} / \hat{B}_4) - B_4 \tilde{\varepsilon} (\dot{\hat{\varepsilon}} / r_\varepsilon - |\xi|) + \tilde{A}_{43} (\xi \theta_{com} - \dot{\hat{A}}_{43} / r_a) \end{aligned} \quad (32)$$

Clearly, \dot{V}_3 is negative semi-definite if the parameter adaptation rules (22)–(25) are chosen

$$\begin{aligned} \dot{V}_3 \leq & -(K_p - A_{43}) K_1 \tilde{\theta}_1^2 + \tilde{B}_4 K_p \tilde{\theta}_1 \xi / \hat{B}_4 - \tilde{B}_4 \dot{\hat{B}}_4 / r_b - \tilde{B}_4 K_1 \xi^2 / \hat{B}_4 \\ & - \tilde{B}_4 K_1^2 \tilde{\theta}_1 \xi / \hat{B}_4 - \tilde{B}_4 \hat{A}_{43} \xi \theta_{com} / \hat{B}_4 + B_4 \tilde{W}^T (S |\xi| - \dot{\hat{W}} / r_w) \\ & - B_4 |\xi| \tilde{\varepsilon} - B_4 \tilde{\varepsilon} (\dot{\hat{\varepsilon}} / r_\varepsilon - 2|\xi|) + \tilde{A}_{43} (\xi \theta_{com} - \dot{\hat{A}}_{43} / r_a) \\ = & -(K_p - A_{43}) K_1 \tilde{\theta}_1^2 \leq 0 \end{aligned} \quad (33)$$

This main result is summarized as below.

Theorem 2 Consider the system (6) with the proposed adaptive control (21) with the parameter adjustment rules (22)–(25). Then the origin is globally asymptotically stable, i.e., $\xi \rightarrow 0$ and $\tilde{\theta}_1 \rightarrow 0$ as $t \rightarrow \infty$ or $\theta_p \rightarrow \theta_{com} = 0$, and $\dot{\theta}_p \rightarrow \dot{\theta}_{com} = 0$ as $t \rightarrow \infty$.

3.3 Backstepping Rotation control using RBF

Consider the rotation subsystem with the friction described by

$$\begin{pmatrix} \dot{\delta} \\ \ddot{\delta} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & A_{66} \end{pmatrix} \begin{pmatrix} \delta \\ \dot{\delta} \end{pmatrix} + \begin{pmatrix} 0 \\ B_6 (C_\delta - f_6) \end{pmatrix} \quad (34)$$

where the estimation of the rotation friction is given by

$$f_6 = \bar{f}_6 / B_6 \quad (35)$$

The unknown nonlinear system function \bar{f}_6 can be parameterized by RBF NN over a compact set, i.e.

$$|\bar{f}_6| \leq \bar{f}_6^* = W^T S + \varepsilon_{\min} = [w_1 \cdots w_n] [s_1 \cdots s_n]^T + \varepsilon_{\min} \quad (36)$$

where W is the optimal weight vector, and ε_{\min} is the NN approximation error bounded by

$$\varepsilon_{\min} < \varepsilon_d \quad (37)$$

If we define the state variables, x_1 and x_2 , by

$$x_1 = \delta, x_2 = \dot{\delta}$$

Then equation (31) is rewritten by

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & A_{66} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + B_6 \begin{pmatrix} 0 \\ C_\delta - f_6 \end{pmatrix}$$

Define the tracking error

$$\tilde{x}_1 = x_1 - \delta_c \quad (38)$$

which leads to the time derivative of \tilde{x}_1 as follows:

$$\dot{\tilde{x}}_1 = \dot{x}_1 - \dot{\delta}_c = x_2 \quad (39)$$

To stabilize the \tilde{x}_1 -dynamics, we choose

$$x_2 = -K_p \tilde{x}_1 \quad (40)$$

Substituting (36) into (35) yields

$$\dot{\tilde{x}}_1 = -K_p \tilde{x}_1$$

The asymptotic stability of the error system can be shown via the Lyapunov stability theory, we select the Lyapunov function

$$V_4 = \tilde{x}_1^2 / 2 \quad (41)$$

$$\dot{V}_4 = \tilde{x}_1 \dot{\tilde{x}}_1 = \tilde{x}_1 (-K_p \tilde{x}_1) = -K_p \tilde{x}_1^2 \leq 0 \quad (42)$$

Barbalat's lemma implies that $\tilde{x}_1 \rightarrow 0$ as $t \rightarrow \infty$, i.e. $x_1 \rightarrow \delta_p$ as $t \rightarrow \infty$

Define a backstepping error

$$\xi = x_2 - (-K_p \tilde{x}_1) = x_2 + K_p \tilde{x}_1 \quad (43)$$

Differentiating ξ gives

$$\dot{\xi} = (A_{66} + K_p) x_2 - B_6 C_\delta - (W^T S + \varepsilon_{\min}) \quad (44)$$

On the other hand, we desire to express $\dot{\tilde{x}}_1$ in terms of ξ and

$$\tilde{x}_1 \text{ by } \dot{\tilde{x}}_1 = x_2 + K_p \tilde{x}_1 - K_p \tilde{x}_1 = \xi - K_p \tilde{x}_1 \quad (45)$$

In order to stabilize the rotation subsystem, we can choose C_δ as

$$C_\delta = [- (A_{66} + K_p) x_2 - \tilde{x}_1 + (W^T S + K_1) \text{sgn}(\xi)] / B_6 \quad (46)$$

Substituting the control law (46) into (44), we have

$$\dot{\xi} = -\tilde{x}_1 + (W^T S + K_1) \text{sgn}(\xi) - \bar{f}_6 \quad (47)$$

To stabilize the \tilde{x}_1 -dynamics and ξ -dynamics, we propose the following Lyapunov function candidate:

$$V_5 = \tilde{x}_1^2 / 2 + \xi^2 / 2 \quad (48)$$

$$\begin{aligned} \dot{V}_5 &\leq -K_p \tilde{x}_1^2 + (W^T S + K_1) |\xi| - |\xi| |\bar{f}_6| \\ &= -K_p \tilde{x}_1^2 - |\xi| (\varepsilon_{\min} - K_1) \end{aligned} \quad (49)$$

where $\varepsilon_{\min} > K_1$. Then \dot{V}_5 is negative semidefinite.

We conclude that $\xi \rightarrow 0$ as $\tilde{x}_1 \rightarrow 0$ ($x_1 \rightarrow \delta_c$) as $t \rightarrow \infty$.

Theorem 3: Consider the system (34) with the proposed backstepping control (46). Then the origin is globally asymptotically stable, i.e., $\xi \rightarrow 0$ and $\tilde{x}_1 \rightarrow 0$ as $t \rightarrow \infty$ or $\delta \rightarrow \delta_c$, and $\dot{\delta} \rightarrow \dot{\delta}_c$ as $t \rightarrow \infty$.

3.4 Adaptive Backstepping Rotation Control Using RBF

This section aims to develop an adaptive backstepping controller with unknown parameters and the uncertain friction. The implicit control C_δ with estimated parameters is proposed as follows:

$$C_\delta = [-(\hat{A}_{66} + K_p)x_2 - \tilde{x}_1] / \hat{B}_6 + (\hat{W}^T S + \hat{K}_1) \text{sgn}(\xi) \quad (50)$$

Substituting the control law (50) into (44) gives

$$\dot{\xi} = (A_{66} - \frac{B_6}{\hat{B}_6} \hat{A}_{66})x_2 + (1 - \frac{B_6}{\hat{B}_6})K_p x_2 - \frac{B_6}{\hat{B}_6} \tilde{x}_1 + B_6(\hat{W}^T S + \hat{K}_1) \text{sgn}(\xi) - \bar{f}_6 \quad (51)$$

The errors of the adaptive parameters \tilde{A}_{66} , \tilde{B}_6 , \tilde{W} and \tilde{K}_1 are described by

$$\begin{aligned} \tilde{A}_{66} &= A_{66} - \hat{A}_{66} \Rightarrow \dot{\tilde{A}}_{66} = -\dot{\hat{A}}_{66} \\ \tilde{B}_6 &= B_6 - \hat{B}_6 \Rightarrow \dot{\tilde{B}}_6 = -\dot{\hat{B}}_6 \\ \tilde{W} &= W - \hat{W} \Rightarrow \dot{\tilde{W}} = -\dot{\hat{W}} \\ \tilde{K}_1 &= \varepsilon_{\min} - \hat{K}_1 \Rightarrow \dot{\tilde{K}}_1 = -\dot{\hat{K}}_1 \end{aligned}$$

The following Lyapunov function candidate is presented in order to find the parameter adaptive laws

$$V_6 = \frac{1}{2} \tilde{x}_1^2 + \frac{1}{2} \xi^2 + \frac{1}{2r_1} \tilde{A}_{66}^2 + \frac{1}{2r_2} \tilde{B}_6^2 + \frac{B_6}{2r_3} \tilde{W}^T \tilde{W} + \frac{B_6}{2r_4} \tilde{K}_1^2 \quad (52)$$

where $r_i > 0$

Then the time derivative of the Lyapunov function V_6 becomes

$$\begin{aligned} \dot{V}_6 &= -K_p \tilde{x}_1^2 + \tilde{A}_{66}(x_2 \xi - \dot{\hat{A}}_{66} / r_1) + \tilde{B}_6(-\xi \tilde{x}_1 - \dot{\hat{B}}_{66} x_2 \xi - K_p x_2 \xi) / \hat{B}_6 \\ &\quad - \tilde{B}_6 \dot{\hat{B}}_6 / r_2 - B_6 \left[\tilde{W}^T (S |\xi| - \dot{\hat{W}} / r_3) \right] + B_6 \tilde{K}_1 (|\xi| - \dot{\hat{K}}_1 / r_4) \end{aligned} \quad (53)$$

If the parameter adaptation rules are chosen as follows:

$$\begin{aligned} x_2 \xi - \dot{\hat{A}}_{66} / r_1 &= 0 \\ (-\xi \tilde{x}_1 - \dot{\hat{B}}_{66} x_2 \xi - K_p x_2 \xi) / \hat{B}_6 - \dot{\hat{B}}_6 / r_2 &= 0 \\ S |\xi| - \dot{\hat{W}} / r_3 &= 0 \\ |\xi| - \dot{\hat{K}}_1 / r_4 &= 0 \end{aligned} \quad (54)$$

Then \dot{V}_6 becomes $\dot{V}_6 = -K_p \tilde{x}_1^2 \leq 0$ implies $\tilde{x}_1 \rightarrow 0$ as $t \rightarrow \infty$, thereby resulting in the updating laws of the four parameters are selected as:

$$\begin{aligned} \dot{\hat{A}}_{66} &= r_1 x_2 \xi \\ \dot{\hat{B}}_6 &= (r_2 / \hat{B}_6) (-\xi \tilde{x}_1 - \dot{\hat{A}}_{66} x_2 \xi - K_p x_2 \xi) \\ \dot{\hat{W}} &= r_3 S |\xi| \\ \dot{\hat{K}}_1 &= r_4 |\xi| \end{aligned} \quad (55)$$

Theorem 4 Consider the system (34) with the proposed adaptive control (50) with parameter update laws (55). Then the origin is globally asymptotically stable, i.e., $\xi \rightarrow 0$ and $\tilde{x}_1 \rightarrow 0$ as $t \rightarrow \infty$ or $\delta \rightarrow \delta_c$, and $\dot{\delta} \rightarrow \dot{\delta}_c$ as $t \rightarrow \infty$.

IV. Simulations, Experimental Results and Discussion

4.1 Adaptive RBFNN Backstepping Self-Balancing Control

This section is devoted to describing simulation results of the proposed adaptive RBF backstepping self-balancing control method. A 3-node RBF NN was chosen for simulations because of on-line calculation requirement. The relevant parameters used for simulations are: $\theta_{com} = 0.025$ rad, $K_p = 655$, $r_b = 0.0001$, $r_w = 0.001$, $r_a = 0.0001$. The control signal is limited to the output interval between -512 and 512. Fig.4 displays the angle response of the inverted pendulum subsystem, indicating that the angle approaches the desired angle about at 0.9 seconds. Fig.5 presents the simulated tracking angle. To illustrate the performance of the adaptive friction compensator. Fig.6 shows the time history of the estimation of the $\hat{W}^T S$; the results revealed that the proposed adaptive friction compensator provides good approximation.

4.2 Adaptive RBFNN Backstepping Rotation Control

This subsection will conduct several simulations to examine the effectiveness of the proposed adaptive RBFNN rotation controller. The parameters of the plant are set as the same in Section 4.1. Fig.7 presents the angle response of the rotation subsystem. We set the desired angle at 0.1 rads. Clearly, the rotation angle reaches to the desired angle about at 0.3 seconds. Figs. 9 and 10 depict respectively the time histories of the backstepping error ξ and the estimate $\hat{W}^T S$.

4.3 Human Driver's Test

This experiment studied whether a human driver can easily steer the designed scooter using the proposed adaptive RBF NN control methods. Figs.10 and 11 show the experimental pictures of the human driving test. Through experimental results, the proposed control methods have been shown useful and effective in steering the vehicle easily and safely.

V. CONCLUSIONS

This paper has presented an adaptive RBF neural network control of a self-balancing two-wheeled scooter driven by two DC motors. The mechatronic method has been used to construct the vehicle, including a differential driving mechanism, a control architecture using digital signal

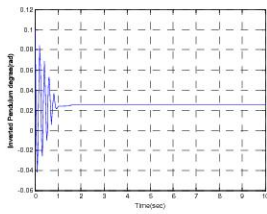


Fig.4. the angle tracking in section 4.1.

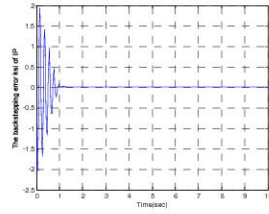


Fig.5 The backstepping error ξ section 4.1

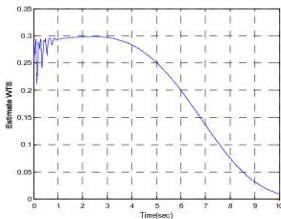


Fig.6. Behavior of the estimate $\hat{W}^T S$ in section 4.1

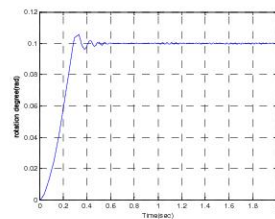


Fig.7. the rotation angle tracking in section 4.2.

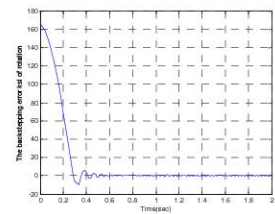


Fig.8. the backstepping error ξ in section 4.2.

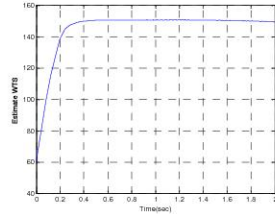


Fig.9. Behavior of the estimate $\hat{W}^T S$ in section 4.2.

processing, a sensor system used for measuring all the wanted feedback signals, such as pitch angle and its rate, yaw angle and rate. The linearized mathematical modeling of the vehicle has been well established in a state-space framework. By decomposing the system into two subsystems: rotation and inverted pendulum, we have synthesized two adaptive RBF NN controllers to achieve self-balancing control and rotation control of the vehicle. Through experimental results, the proposed controllers have been shown useful and effective in providing appropriate control actions to steer the vehicle at slow speeds in expected manners. An important topic for future research will be to construct an adaptive fuzzy controller for the scooter at high speeds, and to conduct several experiments to examine their efficacy.

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Fig.10 Experimental picture of the human driver's self-balancing control.

Fig.11 Experimental picture of the human driver's rotation control.

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